Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 1	DAILY TUTORIAL SHEET 4

76.(ABD) Number of triangles $= {}^{3p}C_3 - p\left({}^3C_3\right) = \frac{p}{2}(3p-1)(3p-1) - p = \frac{9p^2(p-1)}{2}$

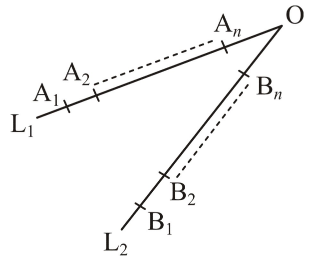
Another approach to solve this is:

Case 1: All 3 points on three different lines: ${}^pC_3 \times \left({}^3C_1\right)^3 = 27\ {}^pC_3$ triangles

Case 2: 2 points on 1 line and 1 point on another: ${}^pC_2 \times \left({}^2C_1\right) \times {}^3C_2 \times {}^3C_1 = 18\ {}^pC_2$ triangles

77.(C) Case I: Triangle contains one vertex as O.

For No. of triangles, follow the steps as given:



Step I : Select 1 point on L_1 in ${}^nC_1$ ways

Step II : Select 1 point on L_2 in ${}^nC_1$ ways

No. of triangles $= \left({}^nC_1 \times {}^nC_1\right) = n^2$

Case II : No triangle has O as its vertex.

Step I : Select 2 points on line L_1 in ${}^nC_2$ ways and one on line L_2 in ${}^nC_1$ ways.

Step II : Select 1 point on line L_1 in ${}^nC_1$ ways and two points on line L_2 in ${}^nC_2$ ways.

No. of ways $= {}^nC_2{}^nC_1 + {}^nC_1{}^nC_2$

$2\frac{n(n-1)}{2}.n = n^2(n-1)$

Total no. of triangles $n^2 + n^2(n-1) = n^3$

78.(C) The required number of triangles is ${}^{15}C_3 - {}^4C_3 = 455 - 4 = 451$

79.(A) The total number of points is 18. From these 18 points we can obtain ${}^{18}C_3$ triangles. However, if all the 3 points are chosen on the same straight line, we do not get a triangle. Therefore, the required number of triangles $= {}^{18}C_3 - 3\left({}^5C_3 + {}^6C_3 + {}^7C_3\right) = 751$

80.(D) Maximum number of triangles $= {}^{3n}C_3 - 3 \cdot {}^nC_3$.

81.(C) $x_1 + x_2 + x_3 = 0$; $x_i \geq -5$; $i = 1, 2, 3$

Let $x_i = y_i + (-5)$ $\Rightarrow$ $y_1 + y_2 + y_3 = 15$; $y_i \geq 0$ $\Rightarrow$ Hence, number of ways $= {}^{15+3-1}C_{3-1} = {}^{17}C_2$

82.(B) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 30$; $x_i \geq 4$

$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 = 6$ where $y_i \geq 0$ [Use $x_i = y_i + 4$]

Total number of ways $= {}^{6+6-1}C_{6-1} = 462$

83.(A) $x + y + z = n, n \geq 3$

As per question, $x \geq 1, y \geq 1, z \geq 1$

Let $x = x_1 + 1; y = y_1 + 1; z = z_1 + 1$ [where, $x_1 \geq 0, y_1 \geq 0, z_1 \geq 0$]

$(x_1 + 1) + (y_1 + 1) + (z_1 + 1) = n$

$x_1 + y_1 + z_1 = n - 3; x_1 \geq 0, y_1 \geq 0, z_1 \geq 0$

Number of ways $= {}^{n-3+3-1}C_{3-1} = {}^{n-1}C_2$

84.(B) $x + y + z + 12 = 0$

$x = -k_1, y = -k_2$ and $z = -k_3; k_1, k_2, k_3 > 0$

$k_1 + k_2 + k_3 = 12$

$k_1 \geq 1, k_2 \geq 1, k_3 \geq 1$ [as $k_i > 0 \Rightarrow k_i \geq 1$]

Number of ways $= {}^{12-1}C_{3-1} = {}^{11}C_2 = \dfrac{11!}{2!9!} = 55$

85.(C) $x + y + z = 100$ where $x \geq 1, y \geq 1, z \geq 1$

$x_1 + y_1 + z_1 = 97; x = x_1 + 1, y = y_1 + 1, z = z_1 + 1$

Number of ways $= {}^{97+3-1}C_{3-1} = {}^{99}C_2 = 4851$ or simply apply ${}^{n-1}C_{r-1}$

86.(B) $a + b + c \leq 8$; $a \geq 1, b \geq 1, c \geq 1$

$a = a'+1, b = b'+1, c = c'+1$; $a'+b'+c' \leq 5; a', b', c' \geq 0$

$a'+b'+c'+d' = 5; a', b', c' \geq 0$; ${}^{5+4-1}C_{4-1} = {}^{8}C_3 = 56$

87.(C) $2b = a + c;\ b + (a + c) = 21$ $\Rightarrow$ $b + 2b = 21$ $\Rightarrow$ $b = 7$

$a + c = 14;\ a \geq 1,\ c \geq 1$

a varies from 1 to 13 = 13

Hence total number of ways = 13

88.(A) $x_1 + x_2 + x_3 + x_4 = 6;\ 1 \leq x_i \leq 6;\ i = 1, 2, 3, 4$

coeff. of x^6 in $(x + x^2 + ... \infty)^4 \equiv$ coeff. of x^2 in $(1 + x + x^2 + . . . \infty)^4$

= coeff. of x^2 in $(1 - x)^{-4} \equiv {}^{4+2-1}C_2 = {}^{5}C_2 = 10$

Another approach

$x_1 + x_2 + x_3 + x_4 = 6;\ x_i \geq 1$. Hence, number of ways = ${}^{6-1}C_{4-1} = {}^{5}C_3 = 10$

89.(A) $x + y + z = 15$

Let $x = x_1 = 0$; $x_1 \geq 0$

$y = y_1 + 1$; $y_1 \geq 0$

$z = z_1 + 2$; $z_1 \geq 0$

$x_1 + y_1 + z_1 = 12$; $x_1 \geq 0, y_1 \geq 0, z_1 \geq 0$

${}^{12+3-1}C_{3-1} = {}^{14}C_2 = \dfrac{14!}{12!\,2!} = 91$

90.(D) $15 < x_1 + x_2 + x_3 \leq 20; x_i \geq 1$ (given $+ve$ roots) $\Rightarrow$ $16 \leq x_1 + x_2 + x_3 \leq 20$

[No. of solutions of $x_1 + x_2 + x_3 \leq 20$] – [No. of solutions of $x_1 + x_2 + x_3 \leq 15$]

$\Downarrow$ $\Downarrow$

$x_1 + x_2 + x_3 + x_4 = 20$ $x_1 + x_2 + x_3 + x_4 \geq 15$

$\uparrow x_4 \geq 0$ $\uparrow x_4 \geq 0$

$y_1 + y_2 + y_3 = 20 - 3 \begin{bmatrix} Use: y_i = x_i - 1; i = 1,2,3 \\ y_4 = x_4 \end{bmatrix}$; $y_1 + y_2 + y_3 + y_4 = 15 - 3$

${}^{17+4-1}C_{4-1}$; ${}^{12+4-1}C_{4-1}$

Number of required solutions $= {}^{20}C_3 - {}^{15}C_3 = 685$

91.(A) $a + b + c + d = 20$

Let $a = 2k_1 + 1;\ k_2 \geq 0, b = 2k_2 + 1;\ k_2 \geq 0, c = 2k_3 + 1;\ k_3 \geq 0, d = 2k_4 + 1;\ k_4 \geq 0$

$2k_1 + 2k_2 + 2k_3 + 2k_4 = 16 \Rightarrow k_1 + k_2 + k_3 + k_4 = 8k \geq 0\ \forall\, i = 1$ to 4

Number of solutions $= {}^{8+4-1}C_{4-1} = {}^{11}C_3 = 165$

92.(A) Total number of non-negative integral solutions of the given equation is same as the number of ways of distributing 100 items among 4 persons such that each person can receive any number of items.

Hence, total number of solutions $= {}^{100+4-1}C_{4-1} = {}^{103}C_3$.

93.(BD) $x_1 + x_2 + x_3 + x_4 \le n$

Let y_4 be a non-negative integer, such that $x_1 + x_2 + x_3 + x_4 + y_4 = n$

Total number of solutions $= {}^{n+4-1}C_{4-1} = {}^{n+3}C_3$

94.(D) Say $x_4 = r$

Total number of solutions $= {}^{20-4r+2}C_2 = {}^{22-4r}C_2$

Total number of solutions $= \sum_{r=0}^{5} {}^{22-4r}C_2$

95.(A) $x_1\, x_2\, x_3 = 60 = 2^2 \cdot 3 \cdot 5$

Say, $x_1 = 2^{a_1} \cdot 3^{b_1} \cdot 5^{c_1}$, $x_2 = 2^{a_2} \cdot 3^{b_2} \cdot 5^{c_2}$ and $x_3 = 2^{a_3} \cdot 3^{b_3} \cdot 5^{c_3}$

$\Rightarrow \quad x_1\, x_2\, x_3 = 2^{a_1+a_2+a_3} \cdot 3^{b_1+b_2+b_3} \cdot 5^{c_1+c_2+c_3}$

$a_1 + a_2 + a_3 = 2$; Total no. of solutions = ${}^4C_2$

$b_1 + b_2 + b_3 = 1$; Total no. of solutions = ${}^3C_2$

$c_1 + c_2 + c_3 = 1$; Total no. of solutions = ${}^3C_2$ $\quad\Rightarrow\quad$ Total no. of solutions $= {}^4C_2 \cdot {}^3C_2 \cdot {}^3C_2 = 54$

96.(B) Twenty pearls $\equiv$ 10 pearls of one color and 10 pearls of another color

Step I: First arrange pearls of same color in $\frac{1}{2}(10-1)! = \frac{1}{2} \times 9!$ ways

Step II: Now arrange pearls of another color in between the arranged 10 pearls in 10! Ways.

No. of ways $= \left(\frac{1}{2} \times 9!\right) \times 10! = 5 \times (9!)^2$

97.(D) $x = 6! \times 6! + 6! \times 6! = 2(6!)^2$ and $y = 5! \times 6!$ $\quad\therefore\quad \frac{x}{y} = 2 \times 6 \Rightarrow x = 12y$

98.(A) Total number of ways $= 4!3!$

99.(A) The 4 men can be seated at the circular table such that there is a vacant seat between every pair of men in $(4-1)! = 3!$ ways.

Now, 4 vacant seats can be occupied by 4 women in 4! ways.

100.(D) First arrange 6 men around the table in $\underline{|5}$ ways. Now, there are 6 gaps between these men. Select any 5 gaps and arrange the women in these gaps in ${}^6C_5 \times \underline{|5}$ ways. So, the required number of ways $\underline{|5} \times {}^6C_5 \times \underline{|5} = \underline{|6} \times \underline{|5}$.